NASA TT F-8772

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N66-11607
(ACCESSION NUMBER) (THRU)
28 1
(PAGES) (CODE)
16
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)

FACILITY FORM 602

GPO PRICE $ _____
CFSTI PRICE(S) $ _____
Hard copy (HC) 2.00
Microfiche (MF) .50
ff 653 July 65

Translation of "Amélioration des prévisions et analyse des mesures Laser pour le S 66."
Centre National d'Etudes Spatiales, Report No. NL/NC/ZK/67, 29 April 1965.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON NOVEMBER 1965

NASA TT F-8772

/SOURCE: (French) National Space Research Center (CNES), Scientific and Technical Division, Section of Mathematics, Report No. ML/NC/ZK/67, 29 April 65./

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IMPROVEMENT OF FORECASTS AND ANALYSIS OF LASER MEASUREMENTS FOR SATELLITE S-66

11607

Abstract: On the basis of a number of laser measurements carried out for the satellite S-66, an attempt is made to furnish improved forecasts by utilizing the elements of the US ephemerides. A second step made it possible to analyze the measurements through a process of differential correction.

Author

A - IMPROVEMENT OF FORECASTS

1 - SELECTION OF ELEMENTS

We had available two series of elements: a) the GSFC bulletins based on NORAD observations from which the preceding forecasts had been prepared; b) the bulletins of the Smithsonian Observatory based on "Minitrack" observations and transmitted by the Meudon Observatory. The latter proved to be better (the times of passage to h or Z (equal) concorded very well with the observations). Consequently, the latter have been utilized.

The elements given at the epoch t_o of the bulletin are:

1 - 1, a_o = mean large semiaxis in km;

1 - 2. e_o = eccentricity;

1 - 3. i_o = inclination in degrees;

1 - 4. M_o = anomaly in degrees;

1 - 5. ω_o = argument of the perigee in degrees;

ω_1 = coefficient of secular term in degrees/day;

1 - 6. Ω_o = right ascension of the ascending node in degrees;

Ω_1 = coefficient of secular tern in degrees/day;

1 - 7. T_o = anomalistic period, i.e., $\frac{\pi^2}{T_o^2} a_o^3 = \mu$ μ = constant of geocentric gravitation;

(1 - 7 and 1 - 1 are equivalent but the datum 1 - 7 is more accurate.)

1 - 8. Drift during this period (friction, lunar-solar or other long - interval terms developed as a function of time) in min/day;

1 - 9 Altitude (above the earth considered as perfect sphere) of the perigee and apogee in km.

1 - 10. Perigeal and apogeal velocities in km/hr.

1 - 11. Geocentric latitude of the perigee in degrees.

2 - CONSTANTS UTILIZED IN THE BULLETIN

Starting from 1 - 9, we can find the equatorial radius of the earth $a(1 - e) - R_e$ = perigeal altitude and/or $a(1 + e) - R_e$ = apogeal altitude.

Starting from 1 - 10, we can find μ: $V^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$ with $V^2 = \mu \frac{1+e}{a(1-e)}$ at the perigee and/or $V^2 = \mu \frac{1-e}{a(1+e)}$ at the apogee.

We found $\mu = 3.98626\ 10^5$ km^3/sec^2 and $R_e = 6378.388$ Km which are the constants of the former international system. We selected $J_2 = 1082.7 \times 10^{-6}$.

3 - ORIGIN OF Ω

The right ascension of the ascending node is computed starting from point γ. We have defined the point γ selected as origin by the Smithsonian bulletin.

At the instant t, we calculated the right ascension of the ascending node by two methods; a) $\Omega = \Omega_0 + \Omega_1 t$, in which Ω_0 and Ω_1 are given by the bulletin; b) $\Omega' = TSG - L$ in which TSG = mean sidereal Greenwich time at the instant t and L = terrestrial longitude of the node positive toward west.

Example for 23 March 65 at 2422 hours 38 sec:

$$\Omega = 314.699$$

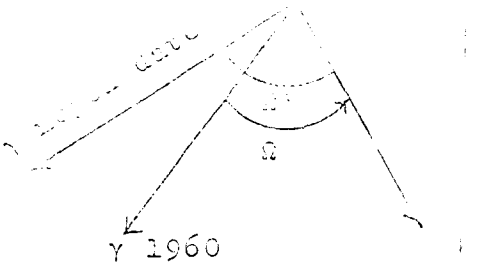
$$\Omega' = 314.769$$

$$\Omega - \Omega' = 0^\circ, 070$$

The difference is due to the fact that TSG, and consequently Ω', is calculated for mean γ of the date whereas Ω is computed from the point γ selected arbitrarily as origin by the Smithsonian.

$\Omega - \Omega'$ corresponds to the precession of the point γ between the

origin of the Smithsonian 1960, 0 and the date of calculation (March 1965).



moyen = mean

This assumes that the law $\Omega = \Omega_0 + \Omega_1 t$ is applied by the Smithsonian. This can be checked by computing the differences between the terrestrial longitudes of the nodes and these differences remain constant and equal to $\Omega_1 \Delta t$ in which Δt is the value of successive intervals.

4 - SIGNIFICANCE OF SECULAR TERMS

We therefore admitted the developments for the angular elements as a function of time and verified that the values of the terms of drift published in the bulletin corresponded to the value of the secular terms due to J_2 calculated theoretically.

$$\begin{cases} \Omega_1 = -\frac{3}{2} n_0 \cos i_0 \, \gamma'^2 \\ \omega_1 = \frac{3}{2} n_0 (-1+5\cos^2 i_0) \, \gamma'^2 \\ n_1 = n_0 \left[1+\frac{3}{2} \gamma'^2 (1-e_0^2)^{1/2} (-1+3\cos^2 i_0) \right] \end{cases}$$

$$\gamma'^2 = \frac{J_2 R_e^2}{2a_0^2 (1-e_0^2)^2} \qquad \begin{cases} J_2 = 1082.7 \cdot 10^{-6} \\ R_e = 6378.388 \end{cases}$$

We find for the anomalistic period: $P_1 = \frac{2\pi}{n_1} =$ [illegible]

It is also possible to calculate the anomalistic period starting from the nodal period derived from the instants of passage at the nodes published in the bulletin; $P'_1 = \frac{2\pi}{n_1 - \omega_1 / 2\pi}$ We find (average of 98 revolutions); P_N = 104 min 48.750 sec, and consequently: P'_1 = 104.75875 min = 6285.5250 sec, and n_1 = 13. 754868 T/day = 0. 9996277 rad/sec.

Note: In the Goddard bulletin, $2\pi/n_1$ is the published anomalistic period (in T/day; the large semiaxis is the mean large semiaxis a_1 in the sense of Kozai, i.e.

$$n_1 = n_o \left[1 + \frac{3}{2} \gamma'^2 (1-e^2)^{1/2} (-1+3\cos^2 i) \right]$$

$$a_1 = a_o \left[1 - \frac{3}{2} \gamma'^2 (1-e^2)^{1/2} (-1+3\cos^2 i) \right]$$

the constants are those of the new international system, i.e. $\mu = 3.98603\ 10^5 \frac{km^3}{sec^2}$ & $R_e = 6378.160$ km and the point of origin γ is the mean point γ of the date.

5-IMPROVED FORECASTS:

We utilized:

5 - 1. The usual forecast program with the data of the Smithsonian;

5 - 2. The extrapolated part of the program of differential correction of Courtois (MA) for calculating the measuring parameters of the osculatory elements determined by the Brouwer theory in assuming that the elements of the bulletin are either mean elements (we then add the terms at long and at short intervals) or mean elements plus long-term intervals developed as a function of time (we then add the short-interval terms).

Moreover, the calculation is carried out with and without drift term.

In both programs, we utilize the Smithsonian elements as is. We consequently also utilize the Smithsonian constants.

The study was carried out with the bulletin of 4 March.

6-COMPARISON WITH OBSERVATIONS

We plotted on the same graph: a) the distances observed with the laser (crosses); b) the distances calculated for the customary forecast program (circles); c) the distances calculated with the terms CP and LP (squares); d) the distances calculated exclusively with the terms LP (dots).

The best results were obtained with the terms LP and CP without drift term. the curves may overlap /"s'ajouter"/ due to a time lag (displacement along the abscissa) and by a displacement along the ordinate which may be due to insufficient knowledge of the position of the perigee.

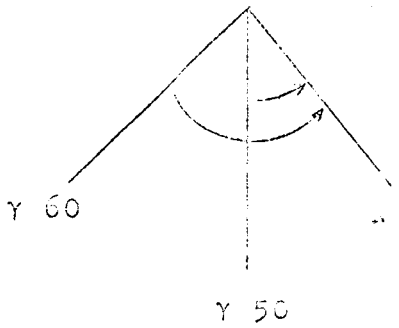
Examination of the bulletins of 12 and 19 February and 4 March shows that the successive ω reduced to the same instant of reference are displaced by some 1/10 degree since the extrapolated ω are always too low /" faible "/. A displacement of ω of 1° produces a difference of distance on the order of 1 km at the instant of measurement under consideration.

7 - CONCLUSION

7 - 1: Maximum use was made of the ephemerides. We found satisfactory agreement on the order of 1 km between the distances measured and those calculated except in regard to the passage of 23 March.

The improvement of the forecasts (Smithsonian bulletin with constants homogeneous with the latter, known point of origin γ, terms with LP and CP) is appreciable, even on the angular parameters where the divergences are on the order of 0.1° (the comparison was made on a measured angular position /" pointe "/ and the position forecast on the basis of the elements of 4 March).

7 - 2: In order to go beyond this, we must utilize the measurements directly in a program of differential correction. The introduction of this program is made with the aid of the elements of the Smithsonian by carrying out two transformations.



7 - 2 - 1: For Ω, the Smithsonian origin is 1960.0 and the CNES origin is 1950.0 We must therefore subtract, from the value of Ω, given in the bulletin, the precession between 1950 and 1960 for 0° 139.

7 - 2 - 2: For a_o, the Smithsonian a'_o is calculated with $\mu' = 3.98628 \times 10^5$ km^3/sec^2 and with $R'_e = 6378.388$km.

The international constants utilized by CNES are

$\mu = 3.98603 \times 10^5$ km^3/sec^2 and $R_e = 6,387.160$ km.

In order to calculate a_o to be utilized in the CNES program, the following conversions must be carried out:

Smithsonian $$n'_1 = n'_o(1+ \varepsilon'(J_2)) \quad (1)$$

CNES $$n_1 = n_o(1+ \varepsilon(J_2)) \quad (2)$$

$$\varepsilon(J_2) = \frac{3}{4} \frac{J_2 R_E^2}{a_o^2 (1-e_o)^2} (1-e_o^2)^{1/2} (-1+3\cos^2 i_o)$$

We desire $n_1 = n'_1$ which gives the value of a_o with the aid of Eq. (2); n'_o is calculated on the basis of the mean anomalistic period of the Smithsonian bulletin $n'_o = \frac{2\pi}{P'_o}$.

7 - 2 - 3: **Numerical application to the bulletin of 4 March 65:**

n'_o = 1.000 1783 ($\rightarrow a'_o$ = 7358.7545 km)

$\varepsilon(J_2)$ = -0.5510015 10^{-3}

n'_1 = 0.996266 10^{-3} rad/sec.

$\varepsilon(J_2)$ = -0.551585 10^{-3}

$n_o = n_1(1- \varepsilon(J_2))$ = 1.00017828

a_o = 7356.6020 km

B - UTILIZATION OF LASER MEASURES OF DISTANCE

Between 23 and 29 March 65, the topocentric distance of satellite S-66 was measured by means of a laser at St. Michel de Provence. The following measurements concerning five passages were transmitted to us:

6 measurements on 23, 3 measurements on 24, 16 measurements on 25, 12 measurements on 28, and 9 measurements on 29 March 65.

By utilizing the orbital parameters of 4 March given by the Smithsonian and with

the aid of a forecast schedule, we plotted the theoretical curve of distance as a function of time for the above five passages. The comparison between the theoretical and the observed curves induced us to eliminate the passage of 23 March since the divergences (O-C) can be ascribed only to errors of measurement or to an inaccurate reading of time (the time data for the passages of 23 and 24 March were not timed on the MSF signals). The passages of 24, 25, 28 and 29 March showed divergences (O-C) on the order of 1 km for distance and 0.5 sec for time (cf. A).

By utilizing the measurements of 24, 25, 28 and 29 March, we attempted to determine the orbit of S-66 and to compare the results obtained with those of the Smithsonian.

This determination was made by means of a program of differential correction of the orbital parameters which utilizes a rather detailed theory of the satellite (see Report No. 24/MA) and enables us to calculate the six osculatory parameters (a, e, i, ω, Ω, M) on the same date together with their drift in time (a_1, e_1, ω_1, Ω_1, M_1, M_2). The solution is obtained by iteration. We started with the values given by the Smithsonian and attempted only to find the osculatory elements at a given date.

The work was done in several steps:

Step 1

We referenced the orbital parameters of 4 March of the Smithsonian to our system of reference (cf. A). After six iterations, we obtained solution 1 (cf. Table A) in which the internal coherence at the convergence is 388 m.

This solution is the most probable orbit, i.e.: $\sum (O-C)^2$ is minimum.

The relation $\sqrt{\frac{\sum(O-C)^2}{m-p}}$ in which m = number of measurements and p = number of parameters to be determined, evaluates the internal coherence of the corrected solution; this is consequently the mean divergence between the theoretical and the measured orbit which may be ascribed equally well to theoretical errors and/or to errors of measurement (along the orbit, on the position of the station, etc.).

LISTING OF MEASUREMENTS UTILIZED

date ⟷ distance (m) ⟷

19644	110	1965	03	25	19	23	04.908	1016068.00	ST-MICHEL
19644	110	1965	03	25	19	23	08.644	1006824.00	ST-MICHEL
19644	110	1965	03	25	19	23	21.628	979772.00	ST-MICHEL
19644	110	1965	03	25	19	23	24.008	974958.00	ST-MICHEL
19644	110	1965	03	25	19	24	30.700	965941.00	ST-MICHEL
19644	110	1965	03	25	19	24	34.061	972507.00	ST-MICHEL
19644	110	1965	03	25	19	24	39.486	980729.00	ST-MICHEL
19644	110	1965	03	25	19	24	41.738	985073.00	ST-MICHEL
19644	110	1965	03	25	19	24	44.133	989932.00	ST-MICHEL
19644	110	1965	03	25	19	24	46.801	995639.00	ST-MICHEL
19644	110	1965	03	25	19	24	52.581	1009026.00	ST-MICHEL
19644	110	1965	03	25	19	25	12.430	1064926.00	ST-MICHEL
19644	110	1965	03	28	19	00	19.674	1026252.00	ST-MICHEL
19644	110	1965	03	28	19	00	26.147	1010056.00	ST-MICHEL
19644	110	1965	03	28	19	00	27.684	1006456.00	ST-MICHEL
19644	110	1965	03	28	19	00	35.707	989267.00	ST-MICHEL
19644	110	1965	03	28	19	00	45.000	972855.00	ST-MICHEL
19644	110	1965	03	28	19	00	49.283	966603.00	ST-MICHEL
19644	110	1965	03	28	19	01	05.879	950540.00	ST-MICHEL
19644	110	1965	03	28	19	02	17.378	1031978.00	ST-MICHEL
19644	110	1965	03	28	19	02	21.012	1042077.00	ST-MICHEL
19644	110	1965	03	28	19	02	25.600	1055515.00	ST-MICHEL
19644	110	1965	03	28	19	02	41.783	1108691.00	ST-MICHEL
19644	110	1965	03	28	19	03	10.374	1221442.00	ST-MICHEL
19644	110	1965	03	29	19	28	33.714	1150307.00	ST-MICHEL
19644	110	1965	03	29	19	28	28.234	1156147.00	ST-MICHEL
19644	110	1965	03	29	19	28	48.614	1140547.00	ST-MICHEL
19644	110	1965	03	29	19	29	29.224	1160065.00	ST-MICHEL
19644	110	1965	03	29	19	29	41.548	1178973.00	ST-MICHEL
19644	110	1965	03	29	19	29	50.508	1196256.00	ST-MICHEL
19644	110	1965	03	29	19	30	18.460	1267845.00	ST-MICHEL
19644	110	1965	03	29	19	30	22.860	1281339.00	ST-MICHEL

25 Mar

28 Mar

29 Mar

When examining the residues, we noted that two of the three measurements of 24 March showed a divergence three times greater than the mean square divergence. This persuaded us to eliminate the measurements of 24 March.

After five iterations, we obtained solution II (cf. Table A), where the internal coherence at the convergence is 80 m.

Five separate points still showed abnormal divergences but were eliminated so that we finally retained a series of 30 measurements for the passages of 25, 28 and 29 March as follows: 12 measurements on 25, 12 measurements on 28, and 8 measurements on 29 March 65 with which we carried out all other tests.

We attempted to obtain a solution by successively combining these three passages two each at a time but there was a deficient convergence of the law of the least squares doubtless due to the insufficiently large number of data furnished for determining the six orbital parameters.

Step 2

In order to arrive at greater precision, we centered the initial date of the parameters in the middle of the period of observation and utilized 27 March at 2400 hours "T.U.".

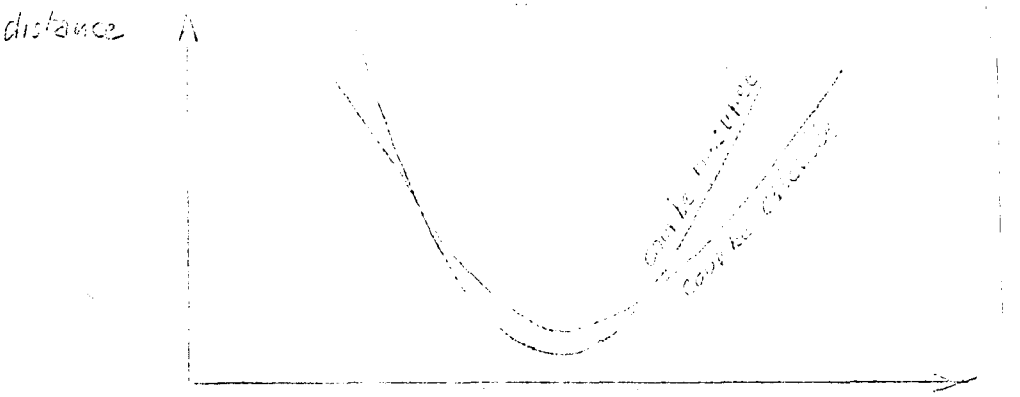
By extrapolating the ephemerides of 18 March of the Smithsonian, we obtained the orbital parameters of 27 March and correlated them to our system of reference.

TABLE A

	SMITHSONIAN PARAMETERS FOR 4 March 65 at 2400 "TU"		SOLUTION I: four passages and 58 measurements on 24, 25, 28, 29 March;		SOLUTION II: three passages and 35 measurements on 25, 28, 29 March;	
a	7358602	m	7358594.1	m	7358583.9	m
e	0.01307		0.013312		0.01289	
i	1.39101	rad.	1.390148	rad.	1.391493	rad.
ω	2.410387	rad.	2.405637	rad.	2.428449	rad.
Ω	5.848563	rad.	5.850868	rad.	5.845967	rad.
M	2.304777	rad.	2.305418		2.284334	rad.
			Internal coherence		Internal coherence	
			388 m		80 m	

In four iterations we obtained solution 1 (Table B), where the internal coherence at the convergence is 10.9 m.

In other respects, the residues have a systematic character for each of the passages. The adjustment is made in accordance with the following diagram:



courbe mesurée = measured curve

courbe calculée = theoretical curve

and we then improved the theory by taking into account the time of aberration heretofore neglected.

In four iterations, we obtained solution 11 (Table B) where the internal coherence at the convergence is 10.1 m.

The residues have the same systematic character and solution 11 is very close to solution 1. From now on, all tests are made by taking into account the time of aberration.

TABLE B

	SMITHSONIAN PARAMETERS ON 27 March at 2400 hours " TU "		SOLUTION I: three passages and 30 measurements on 25, 28, 29 March (time of aberration neglected)		SOLUTION II: three passages and 30 measurements on 25, 28, 29, March (including time of aberration)	
a	7358589.7	m	7358595.4	m	7358585.4	m
e	0.01307		0.012494		0.129503	
i	1.390957	rad.	1.391237	rad.	1.391254	rad.
ω	1.395510	rad.	1.407165	rad.	1.407123	rad.
Ω	5.415517	rad.	5.414782	rad.	5.414787	rad.
M	3.277112	rad.	3.265295	rad.	3.265330	rad.
			Internal coherence 10.9 m		Internal coherence 10.1 m	

TABLE C

	SMITHSONIAN PARAMETERS on 27 March at 2400 hours " TU "	SOLUTION I: three passages and 30 measurements on 25, 28, 29 March (without drift along a).	SOLUTION II: three passages and 30 measurements on 25, 28, 29 March (including drift along a)
a	7358589.7 m	7358589.4 m	7358586.4
e	0.01307	0.012989	0.012976
i	1.390957 rad.	1.391121 rad.	1.391121 rad.
ω	1.395516 rad.	1.396353 rad.	1.396487 rad.
Ω	5.415527 rad.	5.414721 rad.	5.414873 rad.
M	3.277112 rad.	3.280093 rad.	3.267446 rad.
a_1	-0.50 m/j	0.	-0.50 m/j
		Internal coherance 11.8 m	Internal coherance 11.4 m

After we obtained more accurate information on the position of the laser of St. Michel, we slightly modified the geographic coordinates of the station as shown below.

	Old coordinates	New coordinates
Longitude west	354°428330	354°428688
Latitude	43°92972	43°929830
Altitude	651 m	658 m

By utilizing these new coordinates, we obtained solution 1 (Table C) in six iterations where the internal coherence at the convergence is 11.8 m, and the residues are distributed as above.

The new bulletin transmitted by the Smithsonian indicating a drift of - 0.50 m/day along the large semiaxis, we introduced this drift as a given datum in the program of differential correction and attempted to determine the six orbital parameters.

After five iterations, we converged toward solution 11 (Table C) where the interneal coherence is 11.4 m and the residues are analogous.

In order to define the orbit of S-66, we adopted solution 11 of Table C. In relation to that of the Smithsonian, this solution shows the following divergences:

Δa	3.3 m
Δe	0.0001
Δi	-0.0001 rad.
$\Delta \omega$	-0.0093 rad.
$\Delta \Omega$	0.0006 rad.
ΔM	0.0096 rad.
$\Delta (\omega + M)$	0.0003 rad.

It should be noted that we did not attempt to determine a_1 with the program because the interval of time during which we had the measurements is too short.

CONCLUSIONS

These few very accurate measurements enabled us to obtain a solution in satisfactory agreement with the values of the Smithsonian. Nevertheless, the conditions for such

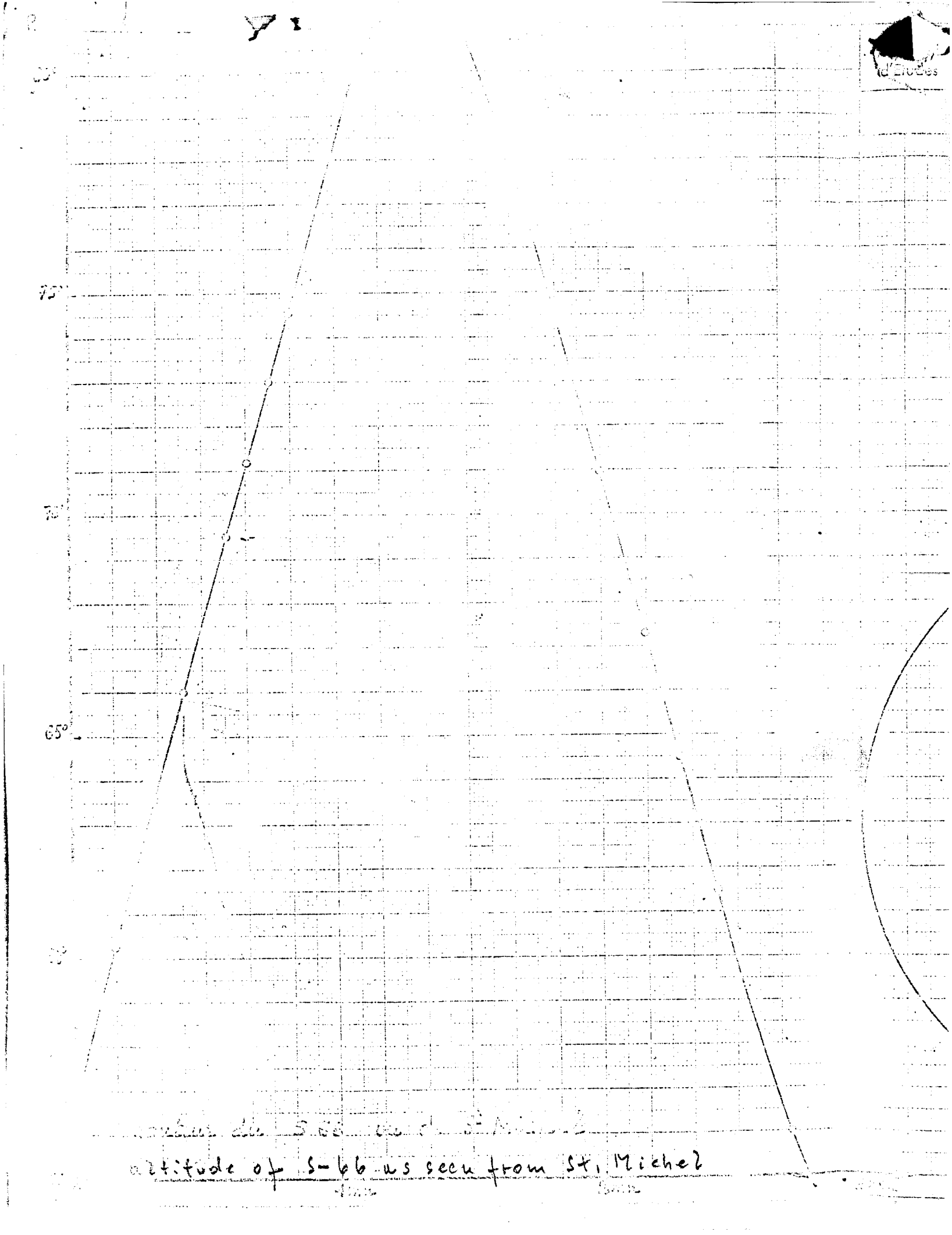
determination are not very favorable: measurements of distance (insufficiently establishing the position of the plane of the orbit and consequently of i and Ω); one single station (same orbital arc of observation); very short interval of time (small orbital arc $\simeq 7^{\circ}$).

In order to be in a better position, it would be necessary:

1) that the parameters of the Smithsonian are interpolated parameters of very accurate measurements (e.g., by means of Baker Nunn) and not extrapolated parameters as above;

2) that there are available not less than two laser stations suitable located for observing two well-separated orbital arcs.

However, in any event, certain parameters (e and ω) will remain poorly established since the orbit is quasi-circular.



Altitude of S-66 as seen from St. Michel

March - time [illegible] : 1900 to 1909.30 hrs

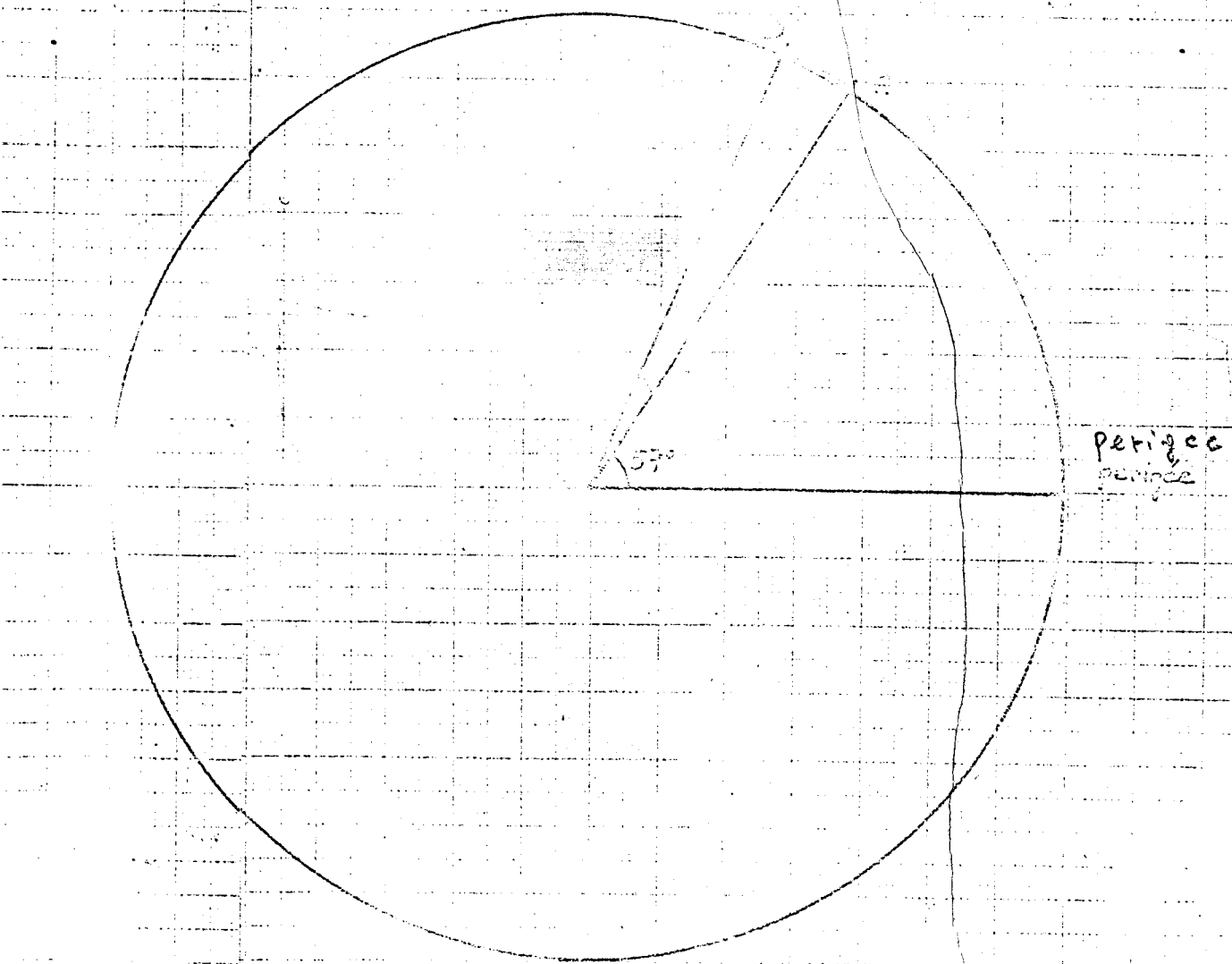
actual measurements
predicted measurements

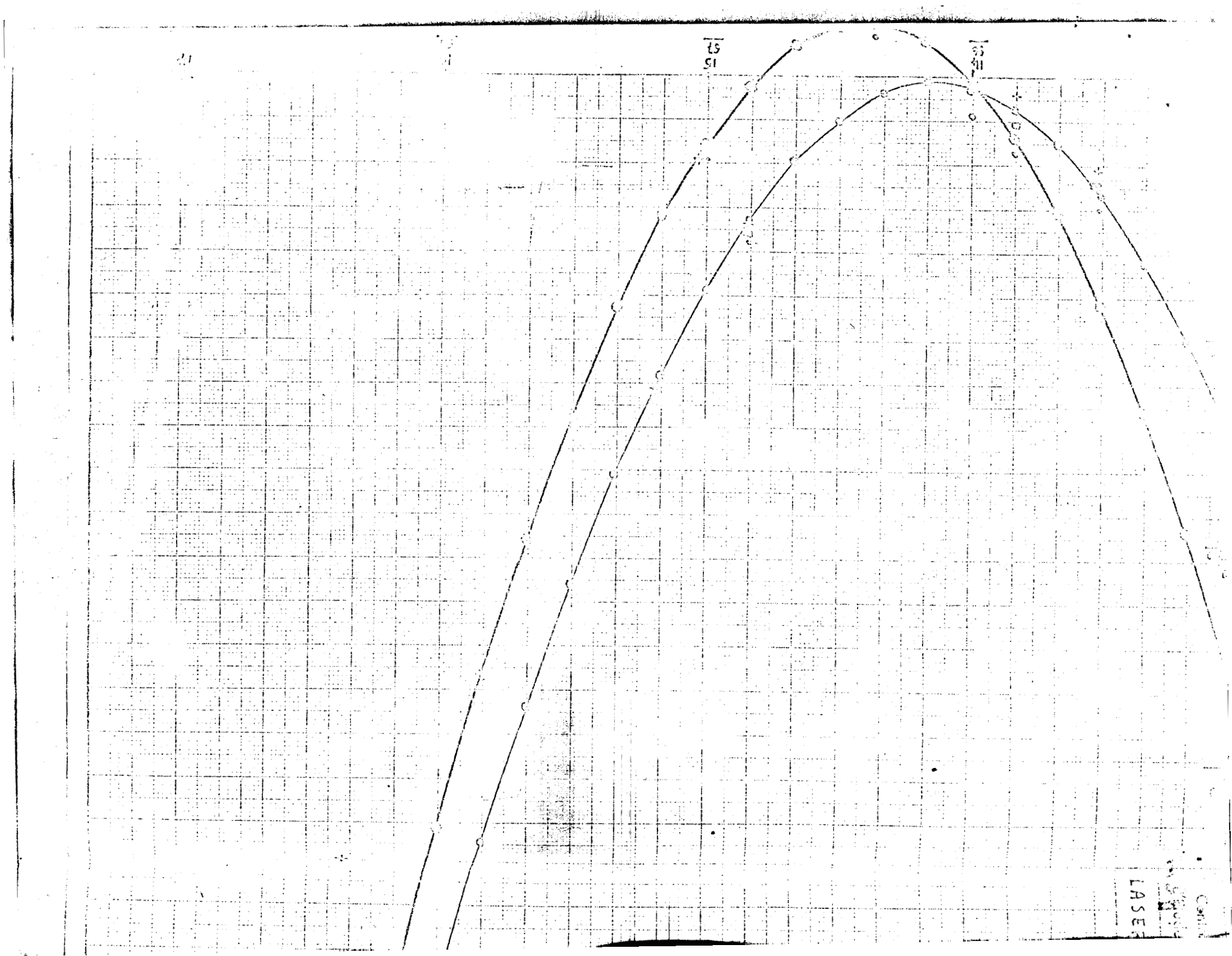
1

orbital arc for which we have laser measurements

[illegible]

$\Delta\Omega = 2°.20$





Centre National d'Etudes Spatiales

LASER 566

24 March — 24 Mars

23 March — 23 Mars

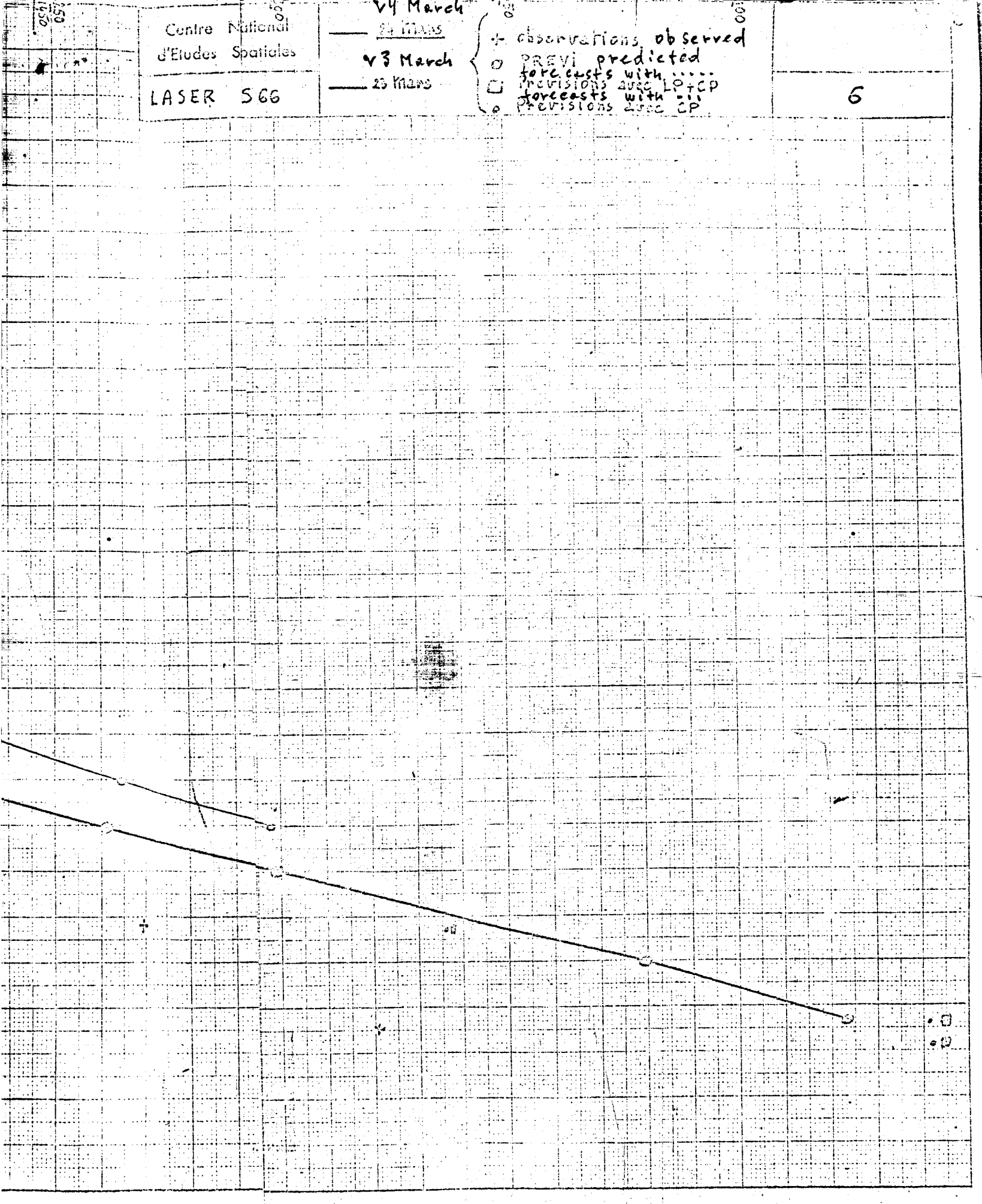
+ observations observed

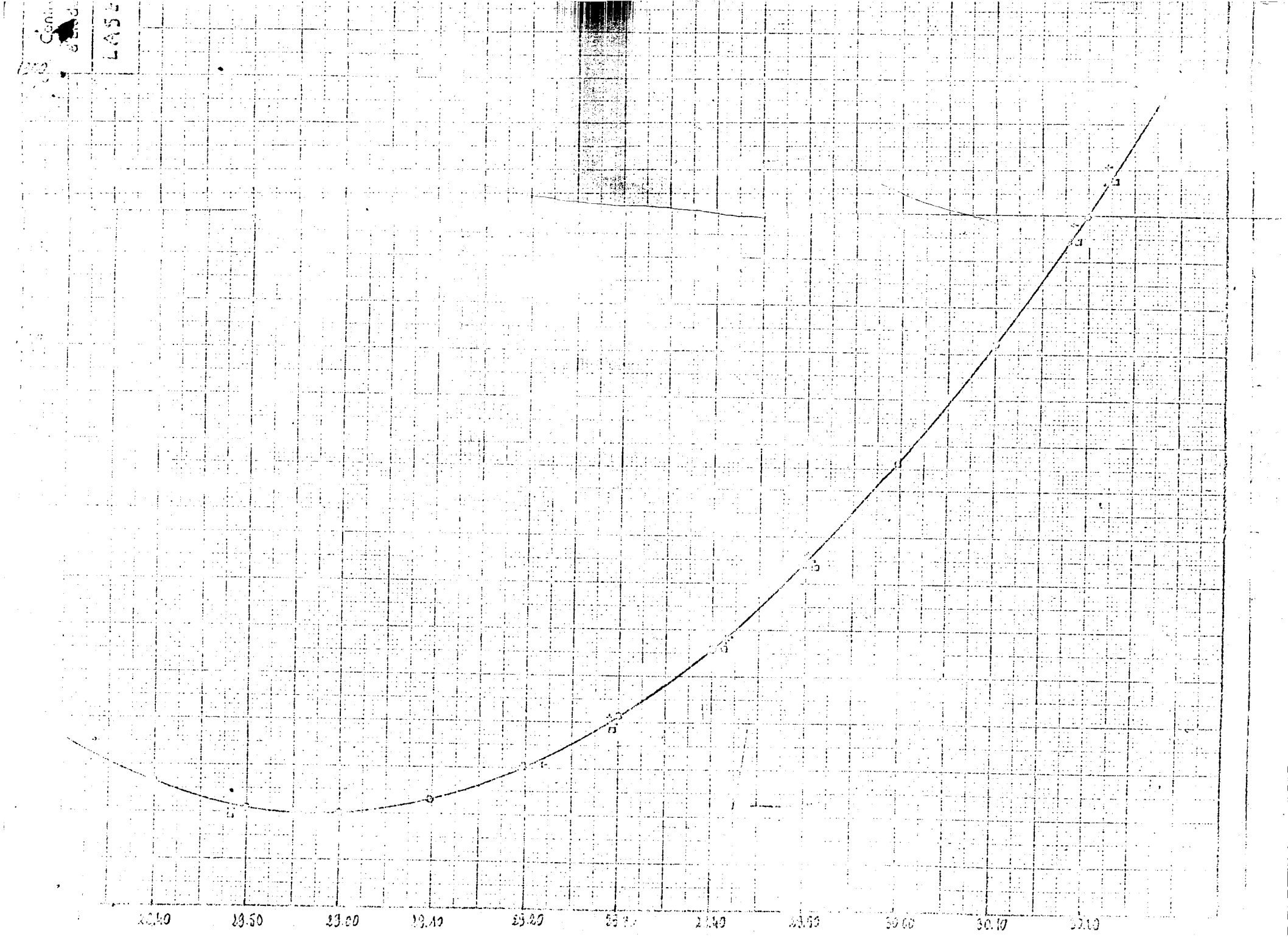
○ PREVI predicted forecasts with .....

□ Prévisions avec LP+CP forecasts with -.-

◇ Prévisions avec CP

6





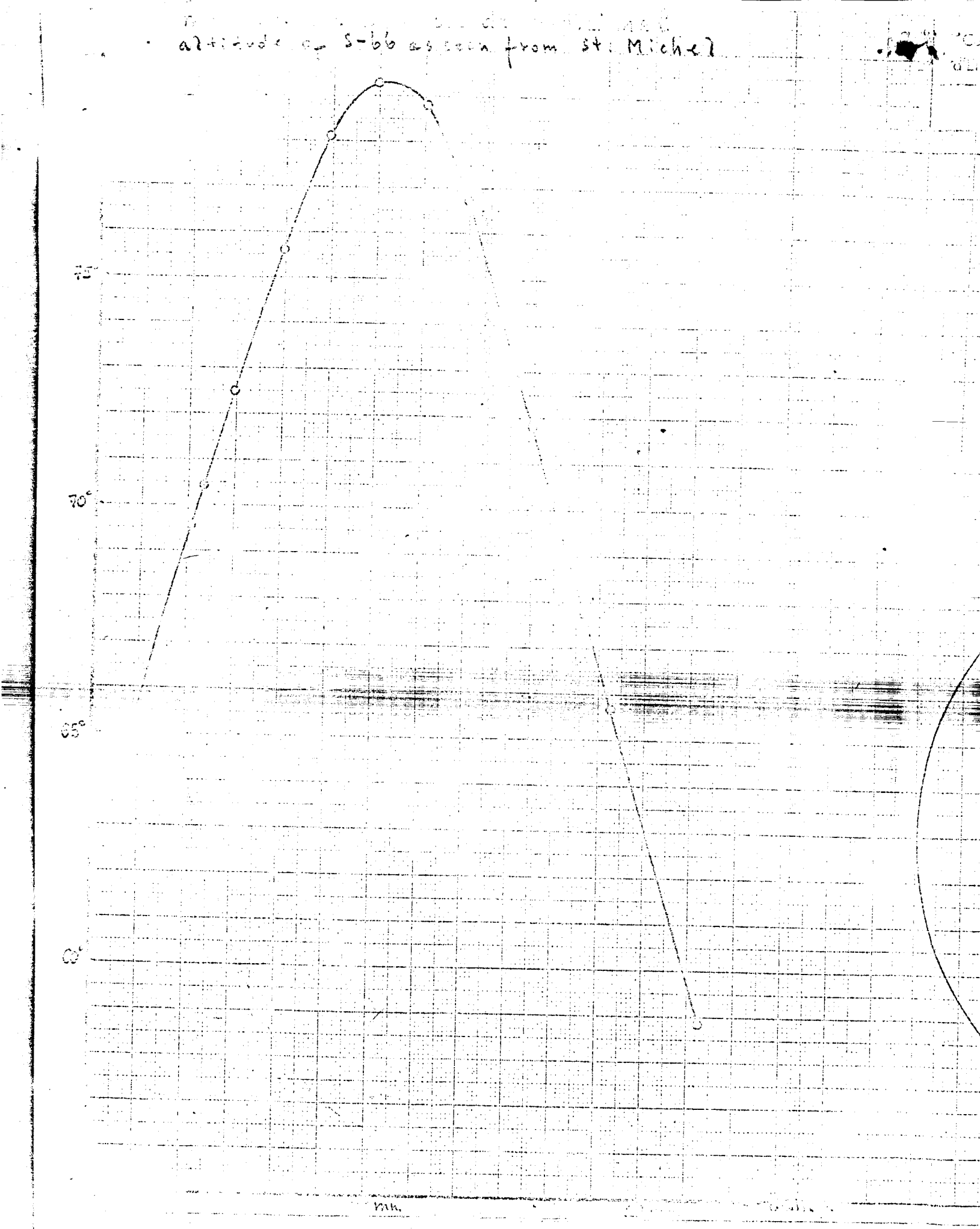
VI Mercer

observed

predicted

5

altitude of S-bb as seen from St. Michel



actual measurements

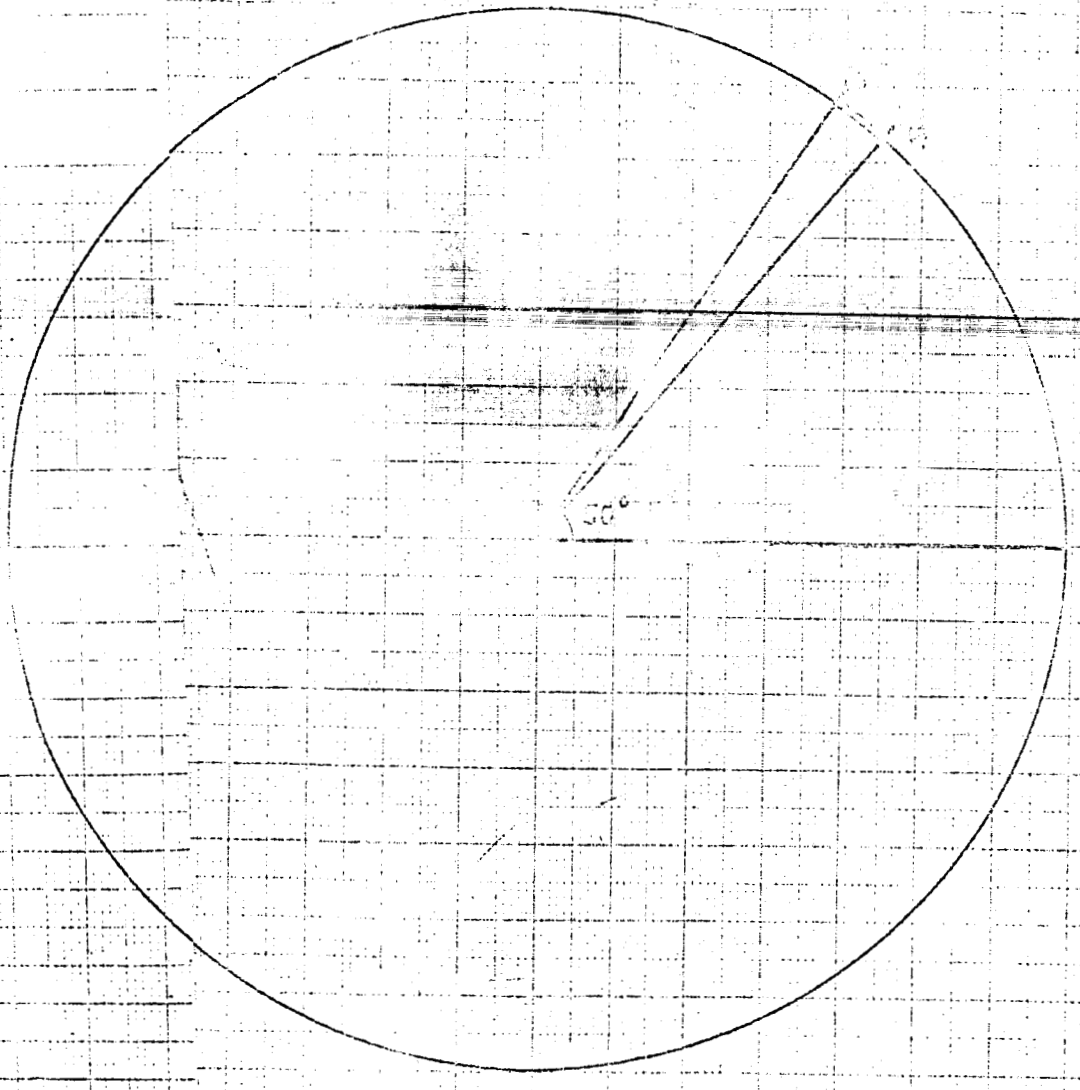
measurements

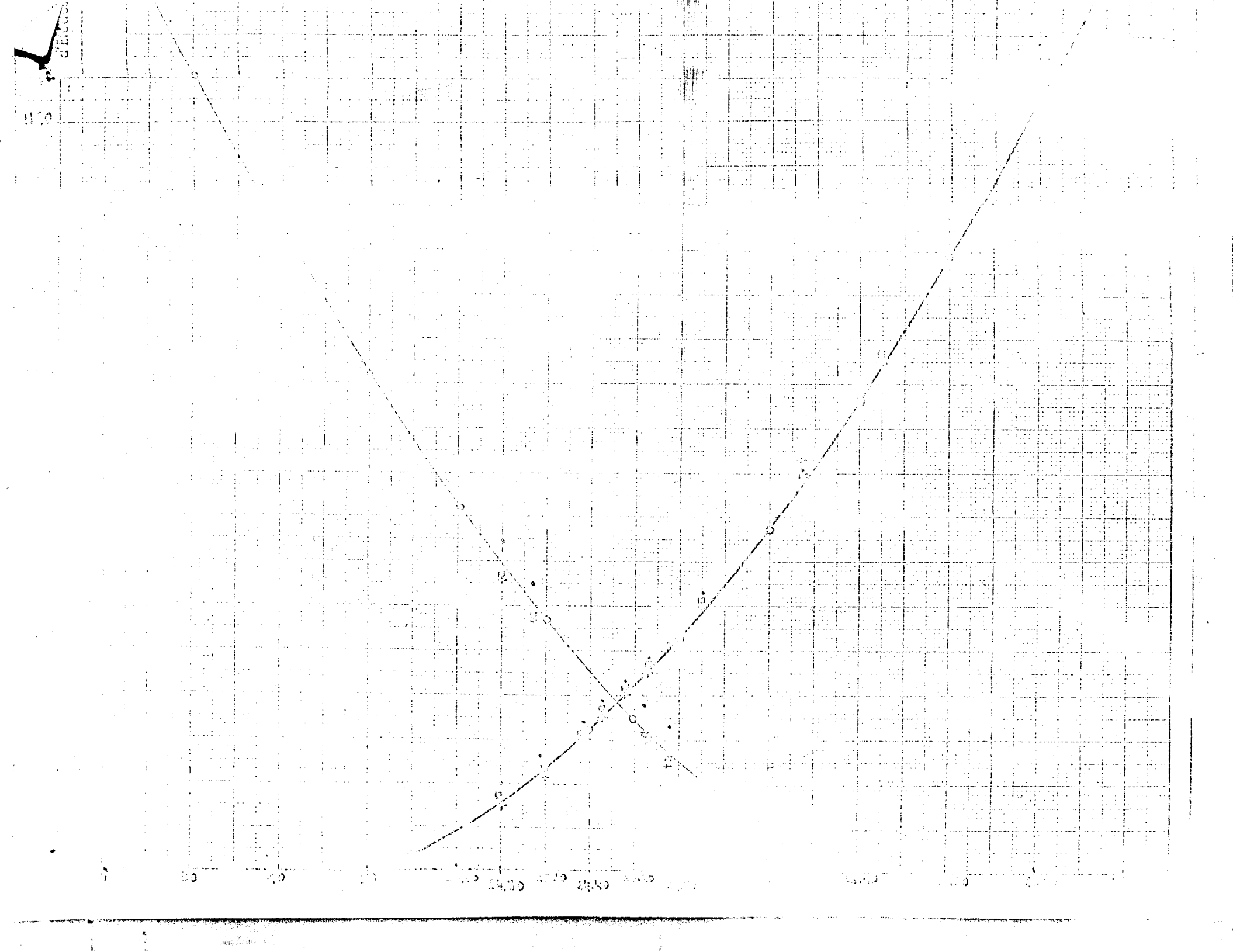
4

orbit assumed as circular

orbital arc for which we have laser measurements

Ω = 40





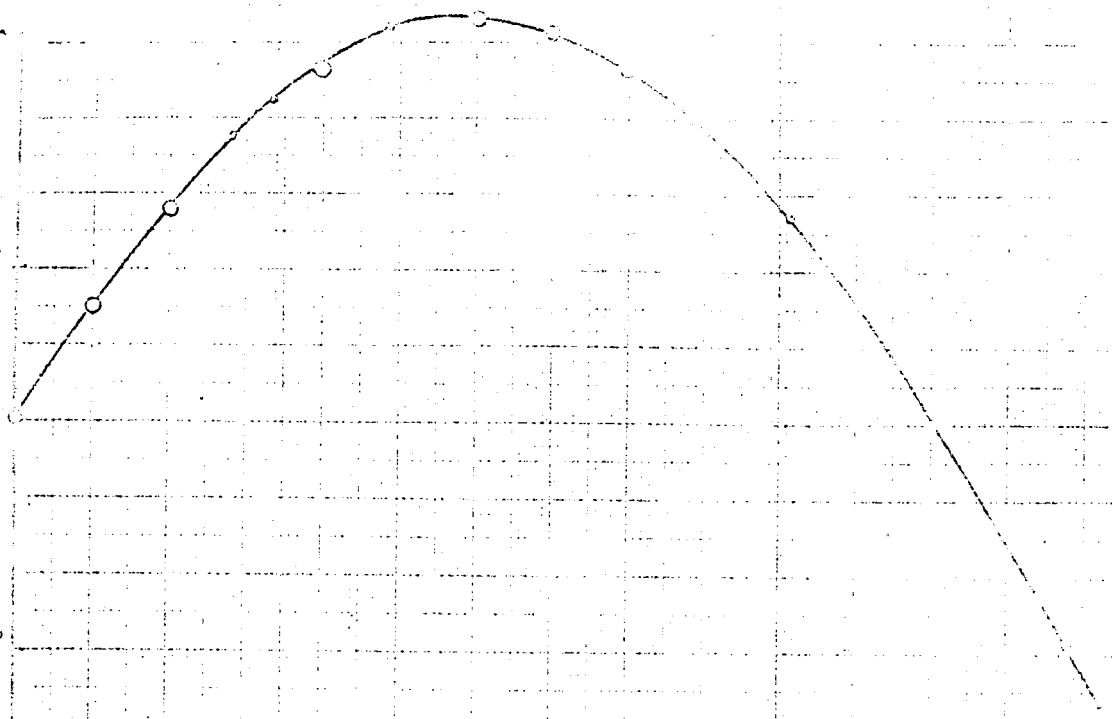
observed

vs March

o MREVI forecast

1250

1300



hauteur du S-66 de St Michel

altitude of S-66 as seen from St. Michel

predicted measurements

2

orbital arc for which we have laser measurements